Distributed cooperative object parameter estimation and manipulation without explicit communication

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Abstract—The paper presents a two stages distributed algorithm for cooperative manipulating an unknown object rigidly grasped by mobile manipulators, in the absence of both a central unit and any explicit information exchange among robots. In the first stage, robots cooperatively estimate the object kinematic and dynamic parameters by properly moving the object or applying specific contact wrenches. In the second stage, the estimated parameters are used in a distributed cooperative algorithm aimed at controlling the object pose while limiting both the squeezing wrenches exerted by the manipulators and the wrench exerted by the environment on the object. Numerical simulations demonstrate the feasibility of the approach.

I. INTRODUCTION

Cooperative manipulators are widely adopted for handling large, heavy or flexible objects. Control and coordination of multi-manipulator systems involved in such tasks require to tackle two sub-tasks [1]: (i) the trajectory tracking of the object both in the absence and in the presence of environmental interactions and (ii) the control of the internal stresses acting on the object due to the contact forces exerted by manipulators. An effective approach to the problem is to consider the manipulators and the grasped object as a whole system, controlled in a centralized manner [2]. Among these object-level control schemes, let us recall hybrid force-position [3], parallel force/position [4] and impedance-based [5] approaches. Impedance control schemes for cooperative manipulators have been employed both for controlling the object/environment interaction and for regulating internal contact forces [6]. In [7] two impedance laws are adopted to confer a compliant behavior to the object and avoid large internal forces. Such a scheme has been extended to aerial manipulators moving a rigid object [8].

Centralized approaches, despite their effectiveness, exhibit some weak points: the central unit can suffer of malfunctions and the system is poorly scalable, since its complexity increases rapidly with the number of robots. Decentralized schemes based on the impedance concept has been adopted in [1], where a distributed impedance at the joint level and a local active force control are used to achieve control of both object internal force and position, and in [9], where authors propose an impedance-based leader-following approach, not requiring explicit communication between the robots and in which each robot uses only its own force, position and velocity measurements, while the object geometry and the fixed contact points are assumed known. A formation control approach for load transportation is proposed in [10], where the contact forces play the role of attractive/repetitive force feedback between the robots. In [11] a fully decentralized motion and force control, without explicit communications of measured signals between the robots, is proposed for a multi-robot system rigidly transporting an object. A decentralized model free second order sliding mode cooperative controller, where each robot exploits only its own state and force measurements, is designed for redundant robots in [12].

Most of the previous approaches require knowledge of object geometry and dynamics as well as the grasping configuration and forces exerted at the grasping points. Such assumptions are often not met in practice and fight against the increasing demand of adopting mobile robots in unstructured environments. A centralized approach to the problem of estimating kinematic and inertial parameters of a grasped load is proposed in [13], where mass, inertia and center of mass of a grasp-less unknown object, pushed by robot fingers, are estimated; while in [14] a least-squares approach is adopted for on-line estimating the object inertial parameters in a robust-to-noise way. In [15], a decentralized strategy for a multi-robot team, whose motion is constrained on a plane, manipulating a load is proposed. It is based on two stages: first, the kinematic and inertial parameters of the load are estimated by resorting to the results in [16], then a robust control is designed. Through a sequence of steps, requiring wireless communication among robots, the algorithm allows all robots to agree on the estimation of all relevant parameter of the load.

In this paper, we propose a distributed algorithm for cooperatively manipulating an unknown object rigidly grasped by mobile manipulators and under the assumptions that each robot has access only to local information. Similarly to [15], a two stages solution is devised: firstly robots cooperatively estimate the object’s parameters, while, in the second stage, a force control strategy, exploiting the estimates of the first stage, is designed with the aim to (i) move the object along a planned trajectory, (ii) regulate the object internal wrenches in order to avoid excessive stresses, (iii) limit the wrench raising during the interaction between the object and the external environment. To this aim, each robot performs an estimate of its own contribution to the overall internal wrench and of the object-environment interaction by exploiting the estimated object parameters and the momentum-based observer proposed in [17]. Differently from existing solutions, no explicit information exchange among robots is considered and, differently from

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The research leading to these results has received funding from Regione Campania (Legge n.5/2002) under the project “Cooperazione multi-robot per chirurgia robotizzata”, Resp. Pasquale Chiacchio.

Research of F. Pierri has been partially supported by the European Community’s Horizon 2020 research and innovation program under grant agreement No. 644271 AEROARMS.

where a planar problem is faced, the whole algorithm is devised in the 3-D space where both position and orientation (forces and torques) are taken into consideration.

II. MATHEMATICAL BACKGROUND

A. Manipulator model

Let us consider a work-cell composed by N mobile manipulators, not necessarily homogeneous, cooperatively manipulating a rigid object (see Fig. 1). The dynamic model of the k-th mobile manipulator can be written as

\[ M_k(q_k)\dot{q}_k + C_k(q_k, \dot{q}_k)\dot{q}_k + F_k + g_k = \mu_k - J^T_k(q_k)\tau_k \]

where \( M_k \in \mathbb{R}^{n_k \times n_k} \) is the joint position (velocity, acceleration) vector, \( \mu_k \in \mathbb{R}^{n_k} \) is the joint torque vector, \( M_k(q_k) \in \mathbb{R}^{n_k \times n_k} \) is the symmetric positive definite inertia matrix, \( C_k(q_k, \dot{q}_k) \in \mathbb{R}^{n_k \times n_k} \) is the centrifugal and Coriolis terms matrix, \( F_k \in \mathbb{R}^{n_k \times n_k} \) is the matrix modeling viscous friction, \( g_k(q_k) \in \mathbb{R}^{n_k} \) is the vector of gravity terms, and \( \tau_k \in \mathbb{R} \) is the vector of interaction wrench between the robot’s end-effector and the object, being \( f_k \in \mathbb{R}^3 \) and \( \tau_k \in \mathbb{R}^3 \) the vectors of forces and moments, respectively. It is assumed that \( n_k \geq 6 \forall k \).

Concerning the local sensor information available to each robot, the following assumption holds.

**Assumption 1:** Each robot is able to localize itself in the common reference frame, \( \Sigma_o \), and each manipulator is equipped with a whist-mounted force/torque sensor in order to measure the interaction forces with the object.

The object dynamics in (2) and (3) can also be expressed in compact form in the world frame \( \Sigma_w \), as

\[ M_o\ddot{v}_o + C_o(v_o) + g_o = G(r)h + h_{env}, \]

where

\[ \begin{bmatrix} mI_3 & O_3 \\ O_3 & I_6 \end{bmatrix} \in \mathbb{R}^{6 \times 6}, \quad C_o = \begin{bmatrix} 0_3 \\ S(w_o)I_3w_o \end{bmatrix} \in \mathbb{R}^{6 \times 1} \]

\[ g_o = m_3 \left[ g^T, \quad 0_3^T \right] \in \mathbb{R}^{6 \times 1}, \]

being \( v_o = [\hat{p}_o^T, \quad \omega_o^T]^T \in \mathbb{R}^6 \), \( I_o \) and \( O_o \) the identity and zero matrices in \( \mathbb{R}^{n \times n} \), respectively, \( 0_m \in \mathbb{R}^{m} \) the column vector in \( \mathbb{R}^m \) with elements all equal to zero. Moreover, it also is \( r = \left[ r_{c1}^T, \quad r_{c2}^T, \quad \ldots, \quad r_{cN}^T \right] \in \mathbb{R}^{3N} \), \( h = \left[ h_1^T, \quad h_2^T, \quad \ldots, \quad h_N^T \right] \in \mathbb{R}^{6N} \), and

\[ G(r) = \begin{bmatrix} G_1, \quad G_2, \quad \ldots, \quad G_N \end{bmatrix} \in \mathbb{R}^{6 \times N} \]

is the grasp matrix with

\[ \begin{bmatrix} I_3 \\ -S(r_{ck})I_3 \end{bmatrix} \in \mathbb{R}^{6 \times 6}. \]

C. Problem formulation and strategy description

The objective is to devise a cooperative control algorithm that allows \( N \) mobile manipulators to rigidly manipulate a common object in the 3-dimensional space, according to the following requirements (R) and constraints (C):

- C1) a central control unit is not present;
- C2) no explicit communication is allowed between robots; the only information they are assumed to share is the cardinality \( N \) of the team;
- C3) the kinematic and dynamic parameters of the object are unknown;

R1) it is required to limit the wrenches exerted by the manipulators on the object;
R2) it is required to limit the wrenches raising during the interaction between the object and the environment.

The devised solution (Fig. 2) is made of two stages. In the first one, the kinematic and dynamic parameters of the load are identified, as described in Section III, by applying a proper sequence of wrenches in such a way to determine a parameter of the load at each step. In the second stage, the estimated parameters are used to design a force control, described in Section IV, that allows to limit both the wrenches exerted by the object and the environment wrench, \( h_{env} \), while still meeting the above constraints.

III. FIRST STAGE: LOAD PARAMETERS ESTIMATION

By referring to Fig. 1, the parameters to be estimated are:

- the mass \( m \) of the object;
- the relative position of the contact point of the \( k \)-th manipulator and the center of mass \( C \) of the object, i.e., \( r_{ck} = \hat{p}_{ck} - \hat{p}_o \);
- the inertia matrix of the object \( I_o \).

According to constraints C1-C3 in Section II-C, differently from [15], the devised procedure does not require that each robot explicit exchanges information with other members of the team. By exploiting the object dynamics and properly choosing the wrench references for the control loop of manipulators, the estimation comes...
as an emergent feature of the system. The following assumptions are made.

Assumption 2: All mobile robots agree on the orientation of the object frame \( \Sigma_o \). Moreover, it is assumed for simplicity’s sake (but it is not mandatory) that the object and robots’ frames have at each time instant the same orientation.

Assumption 3: The load identification stage occurs in the free space, i.e., \( h_{enw} = 0 \) in (2), (3) and (4).

Assumption 4: It is assumed that the control layer of each robot allows to either control the end-effector motion or the interaction wrench. In general, we assume that hybrid force/position is made available by robots [18].

A. Estimation of the object mass

In order to estimate the mass \( m \), the idea is to have the \( N \) manipulators to cooperate in order to keep the orientation of the object constant over time, while exerting at steady state the same force on the object (i.e., \( f_k = f \), \( \forall k \)). Based on Assumption 4, it is assumed that, for each robot, the contact force and the orientation of the end-effector are controlled (i.e. hybrid force/position control is adopted), with the contact forces set as:

\[
f_k(t) = -k^m_{1,p} \dot{p}_k + k^m_{2,p}(p_k(t_0) - p_k(t)), \quad (8)
\]

with \( k^m_{1,p} \) and \( k^m_{2,p} \) positive scalar gains, and \( p_k(t_0) \) is the initial position of the \( k \) th end-effector. Concerning the orientation of each end-effector, it is regulated to the initial configuration \( R_k(t_0) \), where \( R_k \) is the rotation matrix describing the orientation of \( \Sigma_k \), so as to obtain a pure translational motion of the load and reaching a steady state configuration. The following theorem holds.

**Theorem 1:** If the wrench in (8) is cooperatively applied by the \( N \) mobile robots, at steady state, the \( k \) th manipulator obtains an estimate of the mass \( m \), as

\[
m = \frac{\sum_i v_z^T f_i}{v_z^T g}, \quad (9)
\]

where \( v_z = [0, 0, 1]^T \in \mathbb{R}^3 \).

**Proof:** By substituting (8) in (2) and considering \( f_{enw} = 0 \) and Assumption 3, it is:

\[
m \dot{p}_c - mg = \sum_i (-k^m_{1,p} \dot{p}_i + k^m_{2,p}(p_i(t_0) - p_i(t))), \quad (10)
\]

If the orientation of the object is kept constant during the motion, the object achieves a pure translational motion and, consequently, it is \( \dot{p}_c = \dot{p}_i \) and \( (p_k(t_0) - p_k(t)) = (p_c(t_0) - p_c(t)), \forall k \) and \( \forall t \geq t_0 \); then, (10) becomes

\[
m \ddot{p}_c + N k^m_{2,p} p_c + N k^m_{2,p}(p_c - p_c(t_0)) = mg. \quad (11)
\]

Taking into account (11) and (8), it is at steady state:

\[
f_k = k^m_{2,p}(p_k(t_f) - p_k(t_0)) = k^m_{2,p}(p_c(t_f) - p_c(t_0)) = \frac{1}{N}mg\quad (12)
\]

with \( t_f \) such as \( \dot{p}_c(t_f) \approx 0 \), \( \forall t \geq t_f \), being dynamics (11) linear, asymptotically stable and forced by a constant input.

Finally, by multiplying from the left both members of the above equation by the constant vector \( v_z \), solving for the unknown \( m \) leads to

\[
m = \frac{N v_z^T f_k}{v_z^T g}, \quad (12)
\]

that completes the proof.

Finally, being force measurements affected by noise, it is more convenient to solve (12) in a least square sense.

B. Estimation of parameter \( r_{ck} \)

The main idea behind the estimation of \( r_{ck} = p_c - p_k \) is to cooperatively keep the center of mass \( C \) of the object in a constant position while all robots exert the same force on the object. At the same time, it is required to change the object orientation in order to excite the unknown parameter. Therefore, as in Section III-A, it is assumed that, for each robot, the contact force and the orientation of the end-effector are controlled. In order to rewrite (2) in terms of the unknown \( r_{ck} \), let us consider that \( p_c = p_k + R_c r_{ck} \) and, as a consequence,

\[
\dot{p}_c = \dot{p}_k + R_c S(w_o)^2 r_{ck} + R_c S(w_o) w_{o}^c. \quad (13)
\]

Thus, folding (13) in (2) with \( f_{enw} = 0 \), it is

\[
m \dot{p}_k + mR_c S(w_o)^2 + S(w_o) w_{o}^c r_{ck} - mg = \sum_i f_i, \quad (14)
\]

that may be conveniently re-casted to

\[
m \dot{r}_{ck} = \sum_i f_i + mg - m \dot{p}_k, \quad (15)
\]

with \( w_{o} = R_o S(w_o)^2 + S(w_o) w_{o}^c \in \mathbb{R}^{3 \times 3} \). By exploiting the estimate of \( m \) made at the previous step and choosing

\[
f_k = \frac{m}{N}g, \quad \forall k, \quad (16)
\]

it holds at each time instant:

\[
\phi_{o}(t) r_{ck} = - \dot{p}_k(t). \quad (17)
\]

that is linear in the unknown \( r_{ck} \). By considering (17) and \( l \geq 2 \) different time instants characterized by different values of \( w_{o} \) and/or \( w_{o}^c \), it is possible to estimate \( r_{ck} \) by resorting to a least square estimation:

\[
r_{ck} = - (\Phi_o^T \Phi_o)^{-1} \Phi_o^T \tilde{p}; \quad (18)
\]

where \( \Phi_o = [\phi_{o}(t_1)^T, \ldots, \phi_{o}(t_l)^T]^T \in \mathbb{R}^{3 \times 3} \) and \( \tilde{p} = [p_{o}(t_1)^T, \ldots, p_{o}(t_l)^T]^T \in \mathbb{R}^{3l} \).
Since regression matrix $\Phi_o$ only depends on the angular velocity of the object and it is independent from the object parameters, the angular velocity can be off-line chosen so as the regression matrix $\Phi_o$ has a small condition number and a large minimum singular value [19].

C. Estimation of the inertia tensor $I_o$

The inertia tensor $I_o$ in the object frame (i.e., $I_o^o$) can be estimated by exploiting (3). Since $I_o^o$ is a symmetric matrix its estimation is equivalent to the estimation of the vector

$$\xi = [I_{ox}^o, I_{oy}^o, I_{oz}^o, I_{xy}^o, I_{xz}^o, I_{yz}^o]^T \in \mathbb{R}^6.$$  \hspace{1cm} (19)

The strategy devised to estimate $\xi$ consists in cooperatively keeping the position of the centroid constant while the orientation of the object is properly modified in order to excite the unknown parameters. The wrench $h_k = [f_k^T, \tau_k^T]^T$ exerted by robot $k$ exploits $m$ and $r_{ek}$ estimated in Sections III-A and III-B, respectively, such as

$$f_k = -m \omega \times g - k_1^o \dot{p}_k,$$

$$\tau_k = \Delta \omega \times \omega - S(f_k^T) r_{ek}^o,$$

where $\Delta \omega$ is a moment equal for each robot (either off-line planned or on-line generated), and $k_1^o$ is a positive scalar gain. By folding (20) in (2) and (3), it is

$$I_o^o \ddot{\omega}_o + S(\omega_o) I_o^o \dot{\omega}_o = N \Delta \omega.$$ \hspace{1cm} (21)

Therefore, being the left-hand side of (21) linear in $\xi$, it is

$$\phi_{I_o}(\omega_o, \dot{\omega}_o) \xi = N \Delta \omega,$$ \hspace{1cm} (22)

where $\phi_{I_o} \in \mathbb{R}^{3 \times 0}$ is such as $\phi_{I_o} \xi = I_o^o \dot{\omega}_o + S(\omega_o) I_o^o \dot{\omega}_o$. Equation (22) is the sought one, since it allows to estimate the unknown parameters by resorting to a least square solution as made in Section III-B.

The term $\Delta \omega$ can be set to any time-varying value known beforehand by robots. A better choice is presented in [20] where $\Delta \omega$ is designed to track a planned angular velocity reference in the presence of unknown dynamics, where for the angular velocity reference holds the same considerations made at the end of Section III-B.

IV. SECOND STAGE: OBJECT MANIPULATION

The kinematic and dynamic parameters estimated in the first stage are adopted in order to allow the robot team to manipulate the object. To this aim, motion synchronization must be ensured and, at the same time, excessive mechanical stresses, both on the grasped object and the manipulation robotic system, should be avoided adopting suitable force control strategies. In detail, the wrenches exerted by the robots on the manipulated object can be classified as:

- **squeezing or internal wrenches**, that do not contribute to the object motion (self-balancing system) and can be exploited to stabilize the grasp without over-stressing the system;
- **external wrenches**, that generate the object motion and balance the force exchanged between the object and the external environment in the case of interaction (i.e. contact with the external environment).

In order to refer the contact wrench of all robots to the object center of mass $C$, let us rearrange the right member of (4) as

$$G(r)h = (1^N \otimes I_6)[(G_1 h_1)^T, (G_2 h_2)^T, \ldots, (G_N h_N)^T]^T = \mathcal{C} \begin{bmatrix} \mathcal{T}_1 \mathcal{N}_1 \tau_1 \mathcal{N}_2 \tau_2 \mathcal{N}_3 \tau_3 \mathcal{N}_4 \tau_4 \mathcal{N}_5 \tau_5 \mathcal{N}_6 \tau_6 \end{bmatrix}^T = \mathcal{C} \mathcal{N},$$ \hspace{1cm} (23)

where $\mathcal{C} = 1^N \otimes I_6 \in \mathbb{R}^{6 \times N6}$ is the constant grasp matrix referred to the center of mass $C$, $\mathcal{N}_k = G_k h_k$ ($\forall k$) is the generalized force transformation between the $k$ th end-effector frame and the object frame $\mathcal{O}_o$, and $\mathcal{N} = \begin{bmatrix} \mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_6 \end{bmatrix} \in \mathbb{R}^{6N}$ is a vector storing the contact wrenches. According to [7], the decomposition in internal and external forces of $\mathcal{N}$ can be obtained as:

$$\mathcal{N} = \mathcal{C} \mathcal{G} \mathcal{N} + \left(I_{6N} - \mathcal{C} \mathcal{G} \mathcal{N}\right) = \mathcal{N}_i + \mathcal{N}_e,$$

where, $\mathcal{N}_i \in \mathbb{R}(\mathcal{C})$ and $\mathcal{N}_e \in \mathbb{N}(\mathcal{C})$ are the external and squeezing wrenches, respectively. The symbol $\dagger$ denote the Moore-Penrose pseudoinverse.

Based on the expression of $\mathcal{C}$ in (23), the explicit form of its pseudoinverse $\mathcal{C}^\dagger$ can be easily computed as

$$\mathcal{C}^\dagger = \frac{1}{N} 1_N \otimes I_6.$$ \hspace{1cm} (25)

From (24) and from the structure of the null projector of $\mathcal{C}$, it is straightforward to compute the local contribution to the internal forces relative to the $k$ th manipulator as

$$\mathcal{N}_{i,k} = \Gamma_k \mathcal{N}_i = \mathcal{N}_i - \Gamma_k \mathcal{G} \left(\mathcal{N}_i + \sum_{j \neq k} \mathcal{N}_j\right),$$ \hspace{1cm} (26)

where $\Gamma_k$ is a selection matrix such as

$$\Gamma_k = \begin{bmatrix} O_6 & \cdots & I_6 & \cdots & O_6 \end{bmatrix} \in \mathbb{R}^{6 \times 6N}.$$ \hspace{1cm} (26)

Equation (26) shows that the local contribution $\mathcal{N}_{i,k}$ depends on the local interaction forces, $\mathcal{N}_i$, and on the force exerted by the other manipulators via term $\sum_{j \neq k} \mathcal{N}_j$. It is worth noticing that, on the basis of Assumption 1, robot $k$ only can access the measure of the interaction wrenches $h_k$ at the contact point, while the estimate of $r_{ck}$ made in Section III-B allows to compute the grasp matrix $G_k$ in (7) and (23) (and, then $\mathcal{N}_i = G_k h_k$). On the contrary, in the absence of a central unit and communication between robots (see constraints C1-C2), the global term $\sum_{j \neq k} \mathcal{N}_j$ is unknown to robot $k$ and has to be estimated as in the following.

A. Squeezing force estimation

To the aim of the local squeezing wrench estimation in (26) and concerning the $k$ th manipulator, the object dynamics (4) can be re-arranged as follows

$$M_o \ddot{\mathbf{v}}_o = \mathbf{f}_k + \sum_{j \neq k} \mathcal{N}_j + h_{env} - C_o(\mathbf{v}_o) - g_o.$$ \hspace{1cm} (27)

where the unknown terms $\sum_{j \neq k} \mathcal{N}_j + h_{env}$, to be estimated, are highlighted. In order to locally obtain a reliable estimate, an online estimator, based on the object momentum [17] is designed. Let us define the object generalized momentum $\mathbf{m} = M_o \mathbf{v}_o \in \mathbb{R}^m$ and the vector $\mathbf{\sigma}_k(t) \in \mathbb{R}^m$ such as

$$\mathbf{\sigma}_k(t) = \mathbf{K} \left( \mathbf{m}(t) - \int_{t_0}^{t} (\mathbf{\omega}(\tau) - \mathbf{\sigma}_k(\tau)) d\tau \right),$$ \hspace{1cm} (28)
where $\beta = g_o + C(v_o)$ and $K$ is a positive-definite matrix gain. It can be shown [17] that the time derivative of $\sigma_k$ is

$$\dot{\sigma}_k(t) = -K\sigma_k(t) + K\left(\sum_{j \neq k} T_j + h_{env}\right). \quad (29)$$

Equation (29) represents a low-pass filter, whose bandwidth can be made arbitrarily large by properly selecting the matrix gain $K$. Therefore, for a sufficiently large matrix $K$, it has been proven in [17] that

$$\sigma_k(t) \approx \sum_{j \neq k} \tilde{T}_j + h_{env}. \quad (30)$$

namely, the variable $\sigma_k$ represents an estimate of the effect on the object of both the environmental forces and the forces exerted by all the manipulators but $k$th. This estimate is adopted to define the controlled variable $\lambda_k$ as

$$\lambda_k = \tilde{T}_k - \Gamma_k \tilde{G}^T (\tilde{G} \tilde{T} + \sigma_k). \quad (31)$$

It is worth noticing that $\lambda_k$ can now be locally computed by robot $k$. On the basis of (30), (31) can be rewritten as $\lambda_k = \tilde{T}_k - \Gamma_k \tilde{G}^T (\tilde{T}_k + \sum_{j \neq k} \tilde{T}_j + h_{env} + \tilde{h}_k)\approx \tilde{T}_k - \Gamma_k \tilde{G}^T (\tilde{G} \tilde{T} + h_{env} + \tilde{h}_k),\quad (32)$ where the null projector of $\tilde{G}$ and (26) were exploited, and

$$\tilde{h}_k = \sum_{j \neq k} \tilde{T}_j + h_{env} - \sigma_k. \quad (33)$$

is the estimation error made by robot $k$. By stacking all the single contributions $\lambda_k$ in $\lambda = [\lambda_1^T, \lambda_2^T, \ldots, \lambda_N^T]^T$ and considering (32), it yields

$$\lambda = \tilde{T} - \tilde{G}^T (\tilde{G} \tilde{T} + h_{env}) - \frac{1}{N} \tilde{h}, \quad (34)$$

where $\tilde{h} = [\tilde{h}_1^T, \tilde{h}_2^T, \ldots, \tilde{h}_N^T]^T$. Therefore, an estimate of the internal forces can be obtained by projecting (34) in the null space of $\tilde{G}$

$$\lambda_{int} = (I_{6N} - \tilde{G}^T \tilde{G})\lambda = \tilde{h}_{int} - \frac{1}{N} (I_{6N} - \tilde{G}^T \tilde{G}) \tilde{h}, \quad (35)$$

where it can be seen that $\lambda_{int} \rightarrow \tilde{h}_{int}$ as soon as $\tilde{h} \rightarrow 0$.

B. Internal Wrench Regulation

The vector $\lambda_k$ can be locally used to globally regulate both the internal ($\tilde{h}_{int}$) and the interaction ($h_{env}$) wrenches to desired values. Let us choose $\lambda_d$, desired value of $\lambda$, as

$$\lambda_d = \tilde{G}^T \lambda_{d} + (I_{6N} - \tilde{G}^T \tilde{G}) \lambda_d = \lambda_{env,d} + \lambda_{int,d}. \quad (36)$$

where, as shown in the following, $\lambda_{env,d} \in \mathcal{R} (\tilde{G})$ and $\lambda_{int,d} \in \mathcal{N} (\tilde{G})$ are, respectively, the desired values for external environmental interaction wrenches and internal wrenches. In addition, based on Assumption 4, a local controller is designed for robot $k$ in such a way to asymptotically drive $\lambda_k \rightarrow \Gamma_k \lambda_d$.

Thus, by virtue of (32) and (34), asymptotically holds

$$\lambda_k = \tilde{T}_k - \Gamma_k \tilde{G}^T (\tilde{G} \tilde{T} + h_{env} + \tilde{h}_k) \approx \Gamma_k \lambda_d \quad (37)$$

Based on (24) and by projecting both members of the second equation in (37) in the null space of $\tilde{G}$, it is

$$(I_{6N} - \tilde{G}^T \tilde{G}) (\tilde{T} - \frac{1}{N} \tilde{h}) = \tilde{h}_{int} - (I_{6N} - \tilde{G}^T \tilde{G}) \frac{1}{N} \tilde{h}$$

$$\approx (I_{6N} - \tilde{G}^T \tilde{G}) \lambda_d = \lambda_{int,d}. \quad (38)$$

In sum, internal wrenches acting on the object can be regulated, in a distributed way, to $\lambda_{int,d}$.

C. Decentralized object impedance

Let us project both members of the second equations in (37) in the range of $\tilde{G}$ (i.e., multiplying both members by $\tilde{G}^T \tilde{G}$), it holds, based on (25),

$$\tilde{h}_e = \tilde{h}_o - \tilde{G}^T h_{env} - \tilde{G}^T \tilde{G} \tilde{h} = - \tilde{G}^T h_{env} - \tilde{G}^T \tilde{G} \tilde{h} \quad \approx \tilde{G}^T \lambda_d \lambda_d = \lambda_{env,d}.$$}

which proves that a suitable choice of $\lambda_{env,d}$ allows to regulate the interaction wrench $h_{env}$.

Based on (36), $\lambda_{env,d}$ can be designed in such a way to enforce the desired dynamics to the object. More in detail, an impedance behavior at object level can be obtained by imposing to the robot $k$ the following desired wrench

$$\lambda_{env,d} = I_{6N} (I_{6N} - \tilde{G}^T \tilde{G})^{-1} (I_{6N} - \tilde{G}^T \tilde{G}) \lambda_d = \lambda_{env,d} = \lambda_{int,d}.$$}

is present, the object acceleration error. $K_d$ is a positive definite gain matrix and $h_{\Delta}$ represents an elastic wrench depending on the chosen orientation parametrization [7].

It is worth noticing that $\lambda_{env,d}$ can be locally computed by robot $k$, since it includes the local measured ($\tilde{h}_k$) or estimated ($\sigma_k$) wrenches and the object dynamics, that, based on (5), can be computed by resorting to the estimates of $m$ and $I_o$ in Sections III-A and III-C and to the robot direct kinematics from which $\omega_o$ is determined.

Based on (40), by recalling (34), the global expression of $\lambda_{env,d}$ can be obtained as

$$\lambda_{env,d} = \tilde{G}^T (\tilde{G} \tilde{T}^T + h_{env} - M_o \ddot{v}_o - K_d \ddot{v}_o - h_{\Delta} - C_o (v_o) - g_o).$$

Under the assumption of perfect tracking of $\lambda$ to $\lambda_d$, by folding (41) and (4) in (39), the following closed loop object dynamics is obtained

$$M_o \ddot{v}_o + K_d \ddot{v}_o + h_{\Delta} = h_{env} + \frac{2}{N} \tilde{G} \tilde{h},$$

where $\ddot{v}_o$ is the object acceleration error.

In [7], it has been proven that (42), in the absence of $\tilde{h}_o$, is asymptotically stable. More in detail, in the absence of interaction with the environment, both the object pose and velocity error converge to zero, while, if $h_{env}$ is present, a different equilibrium point is reached. The last term in (42) is a disturbance term whose magnitude depends on the approximation error made in (30).
V. SIMULATIONS

In this section, simulation results relative to the devised strategy are presented. A setup characterized by $3$ ($N = 3$), 8-DOFs ($n_i = 8$, $i = 1, 2, 3, 4$) Comau SmartSix serial chain manipulators (6-DOFs) mounted on a holonomic mobile base (2-DOFs) able to move in the X-Y plane is considered (see Fig. 3). The mission consists in manipulating a cylindrical object from an initial position $[0, 0, 0.3]^{T}$ m to a final one $[-3, 0, 0.25]^{T}$ m while keeping the orientation constant. The cylinder is characterized by radius 1 m, height 0.1 m, $m = 15$ Kg and $I_y = \text{diag}\{1.2, 1.2, 0.6\}$ Kg/m$^2$. Based on the configuration shown in Fig. 3, it also exists $r_{e1} = [0.5, 0, 0]^{T}$ m, $r_{e2} = [0, -0.5, 0]^{T}$ m and $r_{e3} = [-0.5, 0, 0]^{T}$ m. Control input in (1) is set as

$$\mu_k = M_k(q_k) y_k + C_k(q_k, \dot{q}_k) \dot{q}_k + F_{a,k} + \tilde{g}(q_k) + J^T_k(q_k) \hat{\alpha}_k,$$

where $y_k \in \mathbb{R}^{8 \times 1}$ is chosen so as to satisfy Assumption 4 and supposed that the manipulator dynamics is known.

A. Load parameter identification

Concerning the identification stage described in Section III, it is assumed that wrench measurements are affected by a normally distributed noise with zero mean, while two sets are considered for the standard deviation:

- low noise: 0.15 N for forces and 0.05 Nm for torques
- high noise: 0.30 N for forces and 0.10 Nm for torques.

For each set and for each parameter to be estimated, $r \pm 20$ estimation trials were carried out. Moreover, let $\alpha_{k,i}$ be the estimation made at trial $i$ by robot $k$ of a generic parameter $\alpha_k$, the performance of the estimation approach is evaluated by considering the average relative error ($\varepsilon_{are}$) and the maximum relative error ($\varepsilon_{mre}$)

$$\varepsilon_{are} = \frac{1}{N} \sum_{i=1}^{N} \frac{\| \hat{\alpha}_{k,i} - \alpha_k \|_2}{\| \alpha_k \|_2}, \quad \varepsilon_{mre} = \max_{k,i} \frac{\| \hat{\alpha}_{k,i} - \alpha_k \|_2}{\| \alpha_k \|_2}.$$

Gains $k_{p,k}, k_{d,k}, k_{f,k}$ in (8) and (20) were set to 60, 300 and 4, respectively. The virtual input $y_k$ in (43) is not detailed here because of the lack of space. However, it is designed according to [18], if hybrid control is required (estimation of $m$ and $r_{e,i}$), or to the direct force/torque control approach with inner position loop as in [21]. Table I reports the estimation results in terms of the statistics introduced above. Simulations show that the estimation process is only slightly affected by noise. It can be noticed that the best result is obtained for the object’s mass $m$, since it is estimated in steady state conditions and it is not affected by the force/motion controller dynamics. In fact, for the mass, the maximum relative error is 0.0058 ($< 0.6\%$) in the case of high measurement noise. Concerning the other two parameters, $r_{e,i}$ and $I_y$, the maximum relative error is 0.076 ($< 8\%$) and 0.09 ($9\%$), respectively, in high noise conditions. The higher error (nevertheless still acceptable) for these parameters is due to the limited bandwidth of the force/motion control loops and to the fact that their estimation exploits acceleration measurements which, generally, are particularly noisy. Therefore, a proper tuning of force/control loops and measurement filtering would further improve the performance of the estimation.

B. Force Control

The virtual input $y_k$ ($\forall k = 1, 2, 3$) in (43) is chosen as

$$y_k = J_k(q_k) M_y^{-1} \left( - K_v \alpha_k + K_f (\dot{\alpha}_k - \lambda_k) - J_k(q_k, \dot{q}_k) \right),$$

where $M_d \in \mathbb{R}^{6 \times 6}$ is a definite-positive diagonal matrix, $K_v \in \mathbb{R}^{6 \times 6}$ and $K_f \in \mathbb{R}^{6 \times 6}$ are positive definite matrix gains, $\lambda_k$ has been defined in (36)-(37) and $\alpha_k$ is the $k$ th end-effector generalized velocity. The elastic wrench $h_{A,k}$ in (37) has been set to $K_{\Delta,ko}$, where $K_{\Delta,ko} \in \mathbb{R}^{9 \times 6}$ is a positive definite stiffness matrix and $e_o$ is the object pose error, defined as $e = [e_{p}, e_{\epsilon}]^{T} = [p_{o,d} - p_{o}, \hat{e}]^{T}$, where $p_{o,d}$ is the desired object position and $\hat{e}$ is the vector part of the unit quaternion extracted from the mutual orientation matrix $R_o R_{o,d}^{T}$, with $R_{o,d}$ representing the desired object orientation. The values of the gain matrices in (44) as well as of the parameters used in (28) for wrench estimation and in (40) for the decentralized object impedance are reported in Table II. In order to ensure a stable grasp, the desired internal forces have been designed in such a way to squeeze the object, and $f_j$ has all its components equal to 11.547 N while $f_s$ and $f_f$ are equal to $-5.774$ N. Desired internal moments are set to zero. At the end of planned trajectory, the object interacts with the environment and a force along the vertical axis arises, which, in turn, has been modeled as a 6DOF damped spring, with stiffness $K_{env} = \text{diag}\{350 I_3 N/m, 50 \sqrt{I_3 Nm/rad}\}$ and damping $D_{env} = \text{diag}\{40 I_3 Ns/m, 40 I_3 Nms/rad\}$. Thanks to the impedance behavior imposed by (37), the manipulators are able to comply with respect to the environmental forces acting on the object, keeping them limited at the expense of larger tracking errors. Simulation results are reported in Figs. 4–7. Figure 4 shows the pose error of the object: the tracking performance is good in the absence of interaction, while during the contact phase an error of about 4 cm along the $z$ axes is experienced, due to the overall compliance of the system. As depicted in Fig. 5, environmental interaction forces grow up as soon as the...
contact is established (the peak around 15 seconds is due to high damping) and, after a certain time, reaches a steady state value of about 2 N. Regarding the force control performance, Fig. 6 reports the external (Figs. 6(a)-6(b)) and internal (Figs. 6(c)-6(d)) wrench errors for the first manipulator. As can be noticed, both force and moments errors quickly converge to zero, and at steady state the error is of the same magnitude of the measurement noise (high noise set in Section V-A). The results related to the other manipulators have similar behavior and are omitted for the sake of brevity. Finally, Fig. 7 depicts the estimation error \( \hat{\sigma}_k \) (\( k = 1, 2, 3 \)) of the forces (Fig. 7(a)) and moments (Fig. 7(b)) exerted on the object, obtained by the first robot via the momentum observer, as demonstrated by the good performance of the controller, the estimated forces and moments are very close to the actual ones, except for the initial instants due to the error on the initial condition and the dynamics of \( \dot{\sigma}_k \) in (29).

Again, the results obtained for the other manipulators have a similar behavior and are omitted for the sake of brevity.

REFERENCES


